

合圖

|||| 1 || 0 0

1 7 0 0

7 1 0

|||| 0 || 0 ||

|||| 1 7 ||

四行相對
有者換之

辛圖

|||| 1 7 ||

|||| 1 7 ||

|||| 1 || 0 0

|||| 1 || 0 0

|||| 1 || 0 0

|||| 1 || 0 0

|||| 1 || 0 0

|||| 1 || 0 0

|||| 1 || 0 0

元圖

以首位谷七圓米一石三斗
變為谷以米三石四十一
一石

|||| 0 ||

|||| 米

0 米

|||| 米

|||| 米

|||| 米

|||| 米

|||| 米

四圓二石四西乃
作九斤得麥二斗

變圖

上四位進粟

|||| 米

|||| 米

|||| 米

|||| 米

|||| 米

|||| 米

|||| 米

|||| 米

下四位進粟

Counting rods

1	2	3	4	5	6	7	8	9
					┌	┐	┑	┒
—	==	≡	≡≡	≡≡≡	└	┘	┙	┚

Questions

- How to use diagrams?
- What is the relation between counting rods and diagrams?
- What does diagrams exactly mean?

數書九章 Shushu Jiuzhang

Mathematical Treatise in Nine Chapters

- The book was written by 秦九韶 Qin Jiushao (1208—1261) in 1247.
- The original edition is lost. There are 3 copies we can see today: One Ming copy and two Qing copies.

The style

- 81 problems
- Problem and Question
- Answer
- Shu術——algorithms without specific numbers(procedures)
- Cao草——procedure with specific numbers (detail of procedures)
- Tu圖——diagrams
- 立術具草，間以圖發之。
- I established procedures and I set up details of the procedures, which were expressed in blank spaces by means of diagrams.

Rod notation

1	2	3	4	5	6	7	8	9	0
			×	○	⊥	⊥	⊥	⊥	○
—	≡	≡	×	○	⊥	⊥	≡	×	○

Comparison

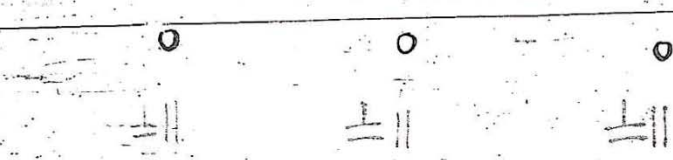
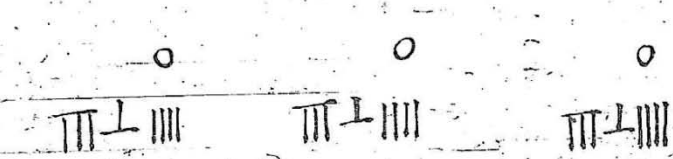
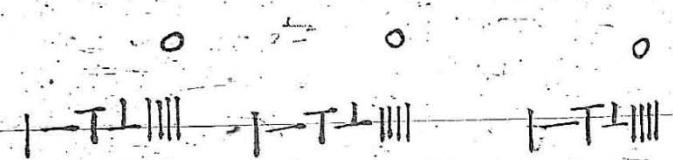
1	2	3	4	5	6	7	8	9	0
					T	TT	TTT	TTTT	
—	==	===	====	=====	⊥	⊥	⊥	⊥	
			X	○	T	TT	TTT	X	○
—	==	===	X	○	⊥	⊥	⊥	X	○

14509

	≡		TTT	
	X	○	○	X

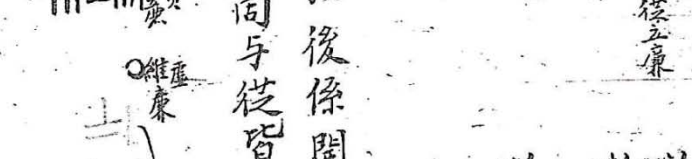
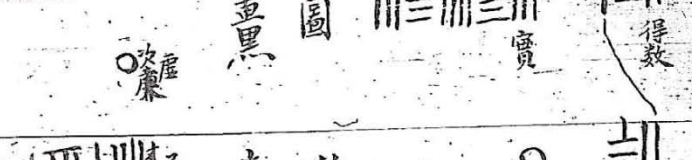
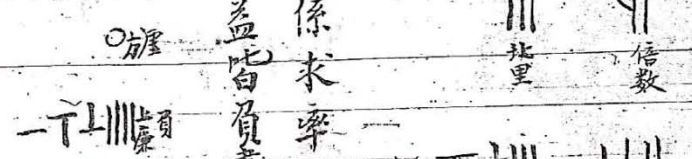
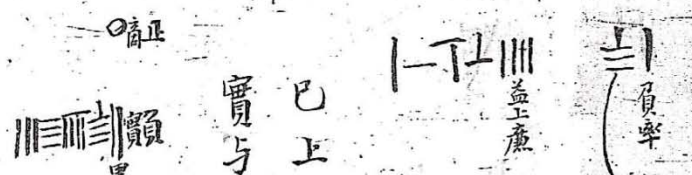
Positive and negative number

- 朱畫用白筭為正，黑畫用黑筭為正。
- Red notations right represent by white rods; black notation right represent by black rods.
- 實與益皆負畫黑，商與縱皆正畫朱。
- Shi and Yi are both negative and written in black; Shang and Zong are both positive and written in red.



以商生陽
得下廉
以商生下
廉得星廉
以商生星
廉得文廉
以商生文廉
入行廉

七



以上係求率圖
實与益比皆負畫黑
以後係開方圖
商与從皆正畫米
以上乘副得次
以次乘下得後上
以上乘副得次實
商重

負率

倍數

得數

從之廉

益上廉
北里

以上乘副得次
以次乘下得後上
以上乘副得次實

已上係求率圖

以後係開方圖

實与益比皆負畫黑

商与從皆正畫米

商

實

商

實

商

實

商

實

商

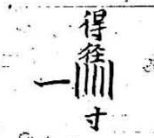
實

商

實

Lines connected numbers

- If two numbers are to be calculated ($+$, $-$, \times , \div), they will be linked by a line.
- The rules are:
- Addition: double lines linked two numbers' heads or tails
- Subtraction: single line linked two numbers' heads or tails
- Multiplication: single line linked one number's head to another number's tail or two numbers' tail
- Division: one dotted line linked one number's head to another number's tail or two numbers' head

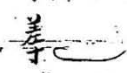


如意益分入
基為口徑

如意差



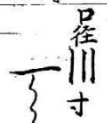
底徑併口徑為和
如意立差損和
得餘



半餘滯中以
差併中為口



口底相乘得上



口底自乘得併



底自乘併上

數書九章卷十二

十一

數書九章三

率 三三

歸 三三

十二

商 三三

斗分餘 三三三

日法餘 三三三

率 三三

歸 三三

斗分餘 三三三

日法餘 三三三

商 一

率 三三

歸 三三

斗分餘 三三三

日法餘 三三三

商 一

商 三三

斗分 三三

日法 三三

天元 三三

歸數 三三

斗分 三三

日法 三三

商 三三

天元 三三

歸數 三三

斗分餘 三三三

日法 三三

天元 三三

歸數 三三

商 三三

Relation between detail of procedures and diagrams

- Diagrams were parallel to the detail of procedures.
- Diagrams were a part of detail of procedures.
- Cao草——detail of procedures

Parallel

- 今仍於各圖立筭求之，以合本術。
- I still set up diagram with rod-notation to look for these (unknowns of the problem), in order to correspond with the original algorithm.
- 此圖照問列位，以後照草運筭。
- In this diagram, positions are placed according to the problem, and in the following, I operate with the counting rods according to the detail of the procedure.
- 俱圖如後。
- I set up diagrams as follows .

- 今列求率開方圖于后。
- Now I list the diagrams following.
- 已上係開三乘方翻法圖，後篇效此。
- Above is the diagrams, and others follow the example.

Parallel

- Diagrams are to record the process of playing rods, and lines in diagrams are to show the relation among numbers and the operations taken on rods.
- But diagrams never display the process of four basic calculations (addition, subtraction, multiplication, division), and in seldom cases diagrams display the extraction of roots.
- Diagrams are another details of procedure (Cao).

Part

- 后圖屢變，每取定率圖數用之。
- Following diagrams always change , and I take the numbers of the determined diagram to use every time.

推求物價

- 問推貨務三次，支物準錢各一百四十七萬貫文。先撥沈香三千五百裹，瑋瑁二千二百斤，乳香三百七十五套；次撥沈香二千九百七十裹，瑋瑁二千一百三十斤，乳香三千五十六套四分套之一；後撥沈香三千二百裹，瑋瑁一千五百斤，乳香三千七百五十套。欲求沈、乳、瑋、瑁、裹、斤、套各價幾何？
- 沈香 A—— Aloe wood
- 瑋瑁 B—— a kind of turtle
- 乳香 C—— frankincense or mastic
- 裹 Guo—— a kind of cover
- 斤 Jin—— measuring unit of weight
- 套 Tao—— another kind of cover

One looks for the prices of things

- Problem: One sold goods three times; the money for the payment of things was exactly 1470000 Guan each time. One first sold 3500 *guo* of A, 2200 *jin* of B, 375 *tao* of C. The following time, one sold 2970 *guo* of A, 2130 *jin* of B, 356 $\frac{1}{4}$ *tao* of C. The last time, one sold 3200 *guo* of A, 1500 *jin* of B, 3750 *tao* of C. One wants to look for how much is respectively the price of a *guo* of A, a *jin* of B, and a *tao* of C?
- 沉香 A—— Aloe wood
- 瑇瑁 B—— a kind of turtle
- 乳香 C—— frankincense or mastic
- 裹 Guo——a kind of cover
- 斤 Jin—— measuring unit of weight
- 套 Tao—— another kind of cover
- 貫 Guan——unit of money

Name	Left	Middle	Right	Detail of the procedure
1				Multiply the middle column by 4.
2				Divide three columns by their greatest common divisors.
3 定率圖 Diagram of the determined lü				<p>三行副置，求之。Put addition rods of three column, and to solve.</p> <p>Multiply the right column by 75; multiply the left column by 15.</p>
4				The right column minus the left column.
5				<p>畢，仍置定圖左行數。When this is finished. Then I still put the numbers of the left column of diagram 3.</p> <p>Multiply the middle column by 75; multiply the left column by 815.</p>
6				The middle column minus the left column.

7	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">左 二三四〇 上三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">中 三三四〇 土三〇 三〇 〇</td> <td style="width: 33%; text-align: center;">右 三三三〇 土三〇 三〇 〇</td> </tr> </table>	左 二三四〇 上三〇 三〇 土三	中 三三四〇 土三〇 三〇 〇	右 三三三〇 土三〇 三〇 〇	<p>畢，仍置定圖左行數。 When this is finished, I again put the numbers of the left column of diagram 3.</p> <p>Divide three columns by their greatest common divisors.</p>
左 二三四〇 上三〇 三〇 土三	中 三三四〇 土三〇 三〇 〇	右 三三三〇 土三〇 三〇 〇			
8 干圖 Diagram gan	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">二三四〇 干上三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">〇三三三〇 三二〇 一三〇 〇</td> <td style="width: 33%; text-align: center;">一三三三〇 沈三〇 三〇 〇</td> </tr> </table>	二三四〇 干上三〇 三〇 土三	〇三三三〇 三二〇 一三〇 〇	一三三三〇 沈三〇 三〇 〇	<p>Multiply the right column by 1815; multiply the middle column by 205.</p>
二三四〇 干上三〇 三〇 土三	〇三三三〇 三二〇 一三〇 〇	一三三三〇 沈三〇 三〇 〇			
9 宮圖 Diagram gong	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">二三四〇 宮上三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">一三三三〇 一三三三〇 三三〇 〇</td> <td style="width: 33%; text-align: center;">二三四〇 三三三〇 三三〇 〇</td> </tr> </table>	二三四〇 宮上三〇 三〇 土三	一三三三〇 一三三三〇 三三〇 〇	二三四〇 三三三〇 三三〇 〇	<p>The right column minus the middle column.</p>
二三四〇 宮上三〇 三〇 土三	一三三三〇 一三三三〇 三三〇 〇	二三四〇 三三三〇 三三〇 〇			
10 干圖 Diagram gan	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">左 二三四〇 上三〇 干三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">中 三三四〇 沈三〇 三〇 〇</td> <td style="width: 33%; text-align: center;">右 一三三三〇 沈三〇 三〇 〇</td> </tr> </table>	左 二三四〇 上三〇 干三〇 三〇 土三	中 三三四〇 沈三〇 三〇 〇	右 一三三三〇 沈三〇 三〇 〇	<p>畢，仍置干圖中行數。 When this is finished, I again put the numbers of the middle column of diagram 8.</p> <p>實如法而一。Divide the top number of right column by 428750.</p>
左 二三四〇 上三〇 干三〇 三〇 土三	中 三三四〇 沈三〇 三〇 〇	右 一三三三〇 沈三〇 三〇 〇			
11	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">二三四〇 上三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">三三四〇 沈三〇 三〇 〇</td> <td style="width: 33%; text-align: center;">三〇 沈三〇 〇 〇</td> </tr> </table>	二三四〇 上三〇 三〇 土三	三三四〇 沈三〇 三〇 〇	三〇 沈三〇 〇 〇	
二三四〇 上三〇 三〇 土三	三三四〇 沈三〇 三〇 〇	三〇 沈三〇 〇 〇			
12 支圖	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">一〇二〇 支三〇 三〇 土三</td> <td style="width: 33%; text-align: center;">三二二〇 三〇 三〇 〇</td> <td style="width: 33%; text-align: center;">三〇 沈三〇 〇 〇</td> </tr> </table>	一〇二〇 支三〇 三〇 土三	三二二〇 三〇 三〇 〇	三〇 沈三〇 〇 〇	
一〇二〇 支三〇 三〇 土三	三二二〇 三〇 三〇 〇	三〇 沈三〇 〇 〇			
13 閏圖	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">閏圖 一〇二〇 〇 三〇 〇 乳三〇</td> <td style="width: 33%; text-align: center;">三〇 〇 三〇 〇 〇</td> <td style="width: 33%; text-align: center;">三〇 沈三〇 〇 〇</td> </tr> </table>	閏圖 一〇二〇 〇 三〇 〇 乳三〇	三〇 〇 三〇 〇 〇	三〇 沈三〇 〇 〇	
閏圖 一〇二〇 〇 三〇 〇 乳三〇	三〇 〇 三〇 〇 〇	三〇 沈三〇 〇 〇			

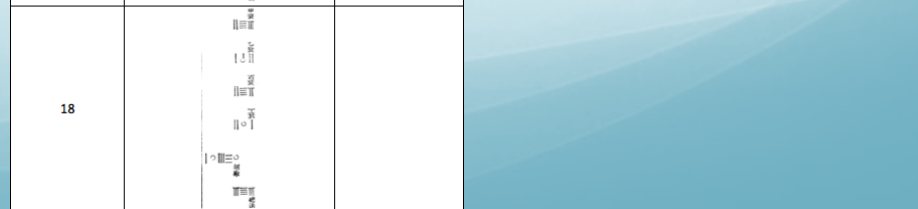
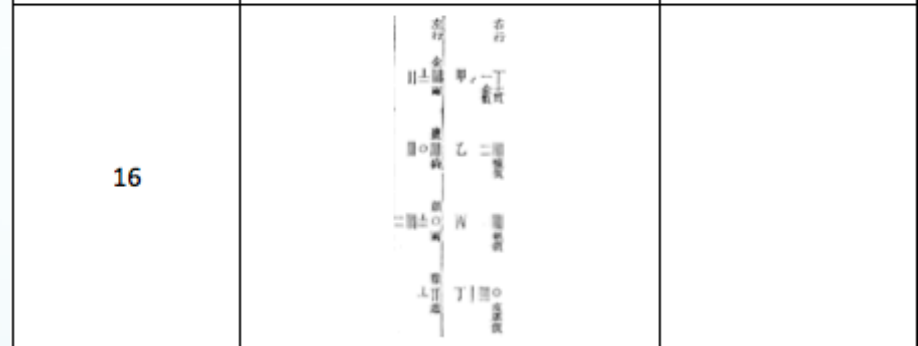
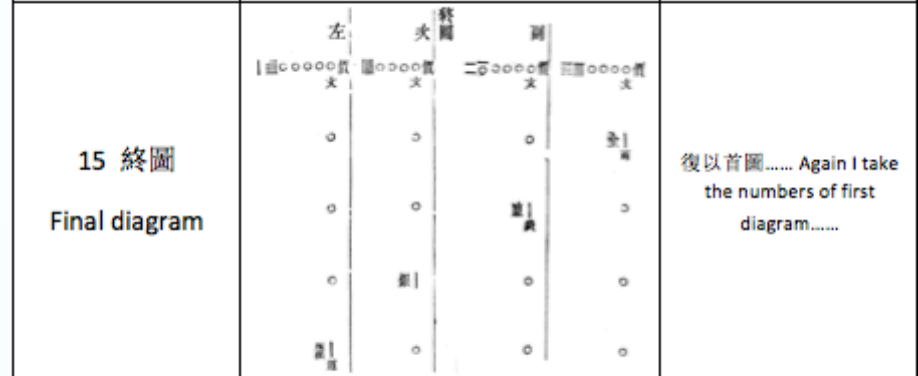
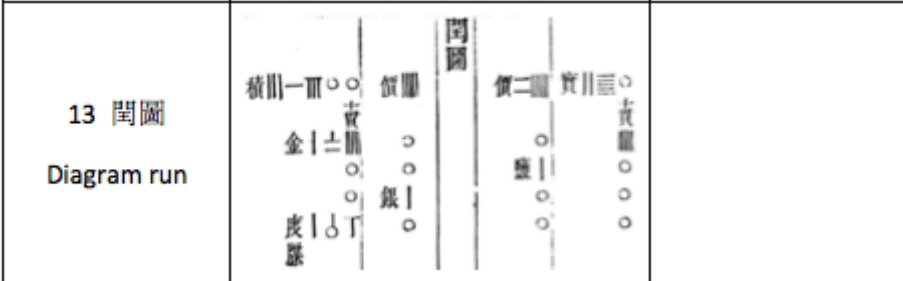
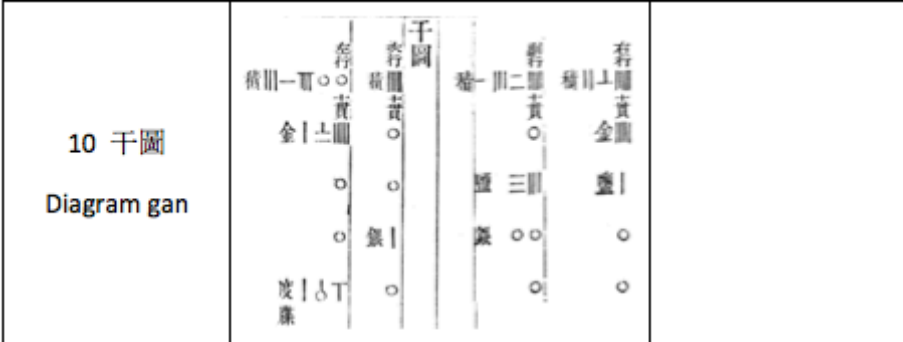
14	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; text-align: center;">三〇 〇 〇 乳三〇</td> <td style="width: 33%; text-align: center;">三〇 〇 三〇 〇</td> <td style="width: 33%; text-align: center;">三〇 沈三〇 〇 〇</td> </tr> </table>	三〇 〇 〇 乳三〇	三〇 〇 三〇 〇	三〇 沈三〇 〇 〇	
三〇 〇 〇 乳三〇	三〇 〇 三〇 〇	三〇 沈三〇 〇 〇			

均貨推本

- 問海舶赴務抽畢，除納主家貨物外，有沈香五千八十八兩，胡椒一萬四百三十包（包四十斤），象牙二百一十二合（大小為合，斤兩俱等），係甲乙丙丁四人合本博到。緣昨來湊本，互有假借，甲分到官供稱：甲本金二百兩，鹽四袋，鈔一十道。乙本銀八百兩，鹽三袋，鈔八十八道。丙本銀一千六百七十兩，度牒一十五道。丁本度牒五十二道，金五十八兩八銖。已上共估值四十二萬四千貫。甲借乙鈔，乙借丙銀，丙借丁度牒，丁借甲金。今合撥各借物歸原主名下，為率均分上件貨物。欲知金、銀、袋、鹽、度牒原價及四人各合得香、椒、牙幾何？
- 沈香 A Aloe wood
- 胡椒 B pepper
- 象牙 C ivory
- 金C Gold
- 盐D Salt
- 银E Silver
- 度牒 F documents used by monk and still could be used as money

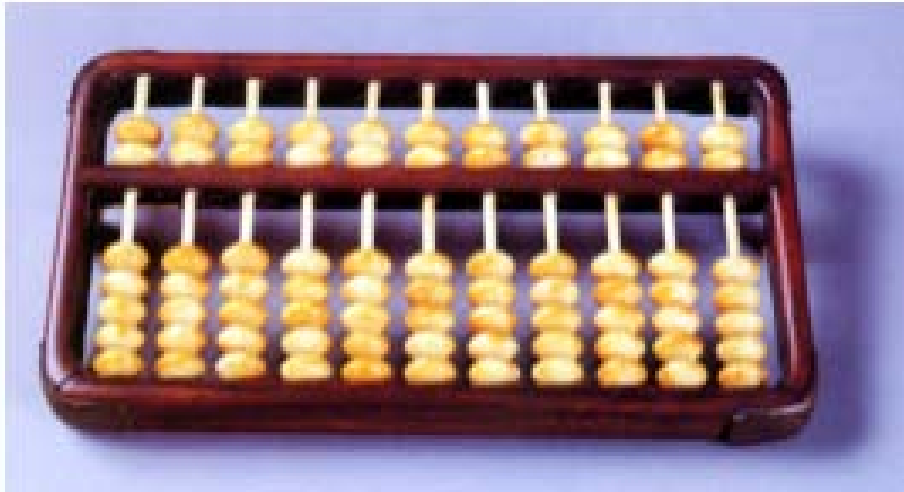
Divide goods equally and look for the capital

- After having conveyed goods by sea boats, except for the goods handed over to the owner's house, there remains 5088 *Liang* of A, 10430 *Bao* of B (1 *Bao*=40 *Jin*) ,212 *He* of C (1*He*=a big ivory + a small one). The remained goods were obtained by four persons (1,2,3,4) , who possess together the capital. Because the four businessmen collected the capital yesterday, they lent and borrowed among themselves. They told the official that the capital of the first person was 200 *Liang* of C, 4 *Dai* of D every row, totally 11 rows; the capital of the second person was 800 *Liang* of E, 3 *Dai* of D every row, totally 88 rows; the capital of the third person was 1670 *Liang* of E, 15 *Dao* of F; the capital of the fourth person was 52 *Dao* of F, 58*Liang* 8*Zhu* of C(1*Liang*=24*Zhu*). It is estimated that altogether the previous (capitals) have a value of 424000 Guan. The first man borrowed D from the second man; the second man borrowed E from the third man; the third man borrowed F from the fourth man; the fourth man borrowed C from the first man. Now all goods should be returned to their masters, the previously mentioned goods being shared in proportion. One wants to know how much the original price of C、 D、 E、 F and how much each of the four persons respectively should obtain?
- 沉香 A Aloe wood 胡椒 B pepper 象牙 C ivory
- 金C Gold 盐D Salt 银E Silver 度牒 F documents used by monk and still could be used as money
- *Jin* , *Liang*—measuring unit of weight
- *Bao*—bag , *Dai*—sack , *He* — used for measuring unit of weight



Conclusion

- What are counting rods?
- Counting rods are not only an instrument for calculation, but an instrument to show some quantitative relation by their positions and notations.
- What are diagrams?
- Diagrams are an instrument to display the process of playing counting rods.
- Diagrams are not aids to calculation, while they are aids to procedure.
- Old procedures and mathematics changed because of making both use of rods and diagrams.



1	2	3	4	5	6	7	8	9	0

Thanks for watching~